

Five best friends watch a movie

Best friends Luke, Leia, Han Solo, Chewbacca, and Darth Vader want to watch a movie. The only movies out in theaters right now are Star Wars, Toy Story, and Frozen. Luke, Leia, Han Solo, and Chewbacca all love Frozen but really don't like Star Wars. At the same time, Darth Vader really wants to watch Star Wars, and he does not want to watch Frozen since he has seen it several times. None of these five friends really want to watch Toy Story, but they will if they must.

Here is what we know:

Voters: Luke, Leia, Han Solo, Chewbacca, and Darth Vader.

Candidates: Star Wars, Toy Story, and Frozen.

Here are the personal rankings:

Luke: Frozen > Toy Story > Star Wars;

Leia: Frozen > Toy Story > Star Wars;

Han Solo: Frozen > Toy Story > Star Wars;

Chewbacca: Frozen > Toy Story > Star Wars;

Darth Vader: Star Wars > Toy Story > Frozen.

Approval voting

Approval voting: Each voter gives 1 point to as many candidates as they want, and the other candidates are given 0 points. The number of points each candidate receives is counted. The candidate with the most points is the winner.

All five friends decide to give 1 point to their top choice, 1 point to their second choice, and 0 points to their last choice.

Fill out the tables below to determine the winner.

Voter Table:

	Star Wars	Toy Story	Frozen
Luke			
Leia			
Han Solo			
Chewbacca			
Darth Vader			

Point Tally Table:

	Number of points
Star Wars	
Toy Story	
Frozen	

Results Table:

	Candidate(s)
First Place	
Second Place	
Third Place	

Alternatively, what if the five friends voted in the following way?

Luke and Leia give 1 point to their top choice, 0 points to their second choice, and 0 points to their last choice.

Han Solo, Chewbacca, and Darth Vader give 1 point to their top choice, 1 point to their second choice, and 0 points to their last choice.

Fill out the tables below to determine the winner.

Voter Table:

	Star Wars	Toy Story	Frozen
Luke			
Leia			
Han Solo			
Chewbacca			
Darth Vader			

Point Tally Table:

	Number of points
Star Wars	
Toy Story	
Frozen	

Results Table:

	Candidate(s)
First Place	
Second Place	
Third Place	

Four friends buy a tub of ice cream

Our community is made up of four friends. Their names are Tom, Jerry, Matilda, and Snow White. The group of friends wants to choose a flavor of ice cream to buy. As a group, they can only choose one flavor. They have four options: chocolate, vanilla, strawberry, and coconut.

Here is a summary of what we know:

Voters: Tom, Jerry, Matilda, Snow White.

Candidates: chocolate, vanilla, strawberry, coconut.

Personal rankings:

Tom: Chocolate > Strawberry > Vanilla > Coconut.

Jerry: Coconut > Strawberry > Vanilla > Chocolate.

Matilda: Vanilla > Chocolate > Strawberry > Coconut.

Snow White: Coconut > Chocolate > Strawberry > Vanilla.

For the case where there are 4 candidates, each voter gives 1 point to their favorite candidate, s points to their second favorite candidate, t points to their third favorite candidate, and 0 points to their fourth favorite candidate, where $1 \geq s \geq t \geq 0$. We can write this voting system as $[1, s, t, 0]$.

Fill out the tables below.

Voter Table:

	Chocolate	Vanilla	Strawberry	Coconut
Tom				
Jerry				
Matilda				
Snow White				

Point Tally Table:

	Number of points
Chocolate	
Vanilla	
Strawberry	
Coconut	

Question 1

Can you find s and t such that the final ranking of candidates is

Chocolate > Strawberry > Coconut > Vanilla?

Recall that $1st0$.

Question 2

Can you find s and t such that the final ranking of candidates is

Strawberry > Chocolate > Vanilla > Coconut ?

Recall that 1st0.

Question 3

Is it possible to have the following final ranking of candidates:

Chocolate > Strawberry > Vanilla > Coconut ?

Why or why not?

Recall that 1st0.

Challenge: Question 4

Is it possible to have the following final ranking of candidates:

Chocolate > Vanilla > Strawberry > Coconut ?

Why or why not?

Recall that 1st0.

The colorful Madrigals

The Madrigal cousins Mirabel, Luisa, Dolores, and Isabella are trying to determine once and for all what the best color is. They have narrowed it down to three colors: blue, purple, and red.

Here is a summary of what we know:

Voters: Mirabel, Luisa, Dolores, and Isabella.

Candidates: blue, purple, and red.

Personal rankings:

Mirabel: Blue > Purple > Red

Luisa: Blue > Red > Purple

Dolores: Red > Purple > Blue

Isabella: Purple > Blue > Red

Comparing Positional Voting Systems

Borda Count: For the case where there are 3 candidates, each voter gives 2 points to their favorite candidate, 1 point to their second favorite candidate, and 0 points to their third favorite candidate. We can write this voting system as [2,1,0].

Fill out the tables below to determine who wins the election under the Borda Count.

Voter Table:

	Blue	Purple	Red
Mirabel			
Luisa			
Dolores			
Isabella			

Point Tally Table:

	Number of points
Blue	
Purple	
Red	

Results Table:

	Candidate(s)
First Place	
Second Place	
Third Place	

Note: There may be ties.

Alternative 1: Each voter gives 1 point to their favorite candidate, $1/2$ point to their second favorite candidate, and 0 points to their third favorite candidate. The number of points each candidate gets is counted. The candidate with the most points is the winner. We can write this voting system as $[1,1/2,0]$. This is a **scalar multiple** of $[2,1,0]$ (recall $[2,1,0]$ is the Borda count).

Fill out the tables below to determine who wins the election under the above voting system.

Voter Table:

	Blue	Purple	Red
Mirabel			
Luisa			
Dolores			
Isabella			

Point Tally Table:

	Number of points
Blue	
Purple	
Red	

Results Table:

	Candidate(s)
First Place	
Second Place	
Third Place	

Note: There may be ties.

Alternative 2: Each voter gives 1 point to their favorite candidate, 0 points to their second favorite candidate, and -1 point to their third favorite candidate. The number of points each candidate gets is counted. The candidate with the most points is the winner. We can write this voting system as $[1,0,-1]$. This is a **translation** of $[2,1,0]$ (recall $[2,1,0]$ is the Borda count).

Fill out the tables below to determine who wins the election under the above voting system.

Voter Table:

	Blue	Purple	Red
Mirabel			
Luisa			
Dolores			
Isabella			

Point Tally Table:

	Number of points
Blue	
Purple	
Red	

Results Table:

	Candidate(s)
First Place	
Second Place	
Third Place	

Note: There may be ties.

An Arbitrary Positional Voting System

For the case where there are 3 candidates, each voter gives 1 point to their favorite candidate, s points to their second favorite candidate, and 0 points to their third favorite candidate, where $1 > s > 0$. We can write this voting system as $[1, s, 0]$.

Fill out the tables below under the above (arbitrary) positional voting system.

Voter Table:

	Blue	Purple	Red
Mirabel			
Luisa			
Dolores			
Isabella			

Point Tally Table:

	Number of points
Blue	
Purple	
Red	

Is there an s where Blue and Purple tie? Recall that $1 > s > 0$.

Is it possible for there to be an s where Red wins the election? Why or why not? Recall that $1 > s > 0$.