

Class Pet

30 students in Namaste Montessori are voting on a class pet. The options are Armadillo, Bee, Cat, Dog, Eagle, Frog, Gecko, and Hamster. The students rank the pets from their favourite to least favourite.

Here are the personal preferences:

10 students vote $A > B > C > D > E > F > G > H$.

10 students vote $B > C > D > E > F > G > H > A$.

10 students vote $C > D > E > F > G > H > A > B$.

Task: Describe as many voting systems as you can, where a Hamster is voted as the class pet.

Four friends buy a tub of ice cream

Our community is made up of four friends. Their names are Tom, Jerry, Matilda, and Snow White. The group of friends wants to choose a flavor of ice cream to buy. As a group, they can only choose one flavor. They have four options: chocolate, vanilla, strawberry, and coconut.

Here is a summary of what we know:

Voters: Tom, Jerry, Matilda, Snow White.

Candidates: chocolate, vanilla, strawberry, coconut.

Personal rankings:

Tom: Chocolate > Strawberry > Vanilla > Coconut.

Jerry: Coconut > Strawberry > Vanilla > Chocolate.

Matilda: Vanilla > Chocolate > Strawberry > Coconut.

Snow White: Coconut > Chocolate > Strawberry > Vanilla.

General positional voting system: Each voter gives 1 point to their favorite candidate, s points to their second favorite candidate, t points to their third favorite candidate, and 0 points to their fourth favorite candidate, where $1 \geq s \geq t \geq 0$. We can write this voting system as $[1,s,t,0]$.

Question 1

Can you find s and t such that the final ranking of candidates is

Chocolate > Strawberry > Coconut > Vanilla ?

The following Voter Table might be helpful:

	Chocolate	Vanilla	Strawberry	Coconut
Tom				
Jerry				
Matilda				
Snow White				

Question 2

Can you find s and t such that the final ranking of candidates is

Strawberry > Chocolate > Vanilla > Coconut ?

The following Voter Table might be helpful:

	Chocolate	Vanilla	Strawberry	Coconut
Tom				
Jerry				
Matilda				
Snow White				

Question 3

Is it possible to find s and t such that the final ranking of candidates is

Chocolate > Strawberry > Vanilla > Coconut ?

Why or why not?

The following Voter Table might be helpful:

	Chocolate	Vanilla	Strawberry	Coconut
Tom				
Jerry				
Matilda				
Snow White				

Challenge: Question 4

Is it possible to find s and t such that the final ranking of candidates is

Chocolate > Vanilla > Strawberry > Coconut ?

Why or why not?

The following Voter Table might be helpful:

	Chocolate	Vanilla	Strawberry	Coconut
Tom				
Jerry				
Matilda				
Snow White				

Voting system: Each voter gives 20 points to their favorite candidate, and 0 points to their second favorite candidate. Each voter gives their second and third favorite candidate some fixed number of points between 0 and 20. The number of points a voter gives to their second favorite candidate is greater than or equal to the number of points that the voter gives to their third favorite candidate. For example, each voter may be asked to give their second favorite candidate 12 points and their third favorite candidate 10 points.

Question 1

How many points can the voters give their second and third favorite candidates so that the final ranking of candidates is

Chocolate > Strawberry > Coconut > Vanilla ?

The following Voter Table might be helpful:

	Chocolate	Vanilla	Strawberry	Coconut
Tom				
Jerry				
Matilda				
Snow White				

Question 2

How many points can the voters give their second and third favorite candidates so that the final ranking of candidates is

Strawberry > Chocolate > Vanilla > Coconut ?

The following Voter Table might be helpful:

	Chocolate	Vanilla	Strawberry	Coconut
Tom				
Jerry				
Matilda				
Snow White				

Question 3

Is the following final ranking of candidates possible:

Chocolate > Strawberry > Vanilla > Coconut ?

Why or why not?

The following Voter Table might be helpful:

	Chocolate	Vanilla	Strawberry	Coconut
Tom				
Jerry				
Matilda				
Snow White				

Challenge: Question 4

Is the following final ranking of candidates possible:

Chocolate > Vanilla > Strawberry > Coconut ?

Why or why not?

The following Voter Table might be helpful:

	Chocolate	Vanilla	Strawberry	Coconut
Tom				
Jerry				
Matilda				
Snow White				

The colorful Madrigals

The Madrigal cousins Mirabel, Luisa, Dolores, and Isabella are trying to determine once and for all what the best color is. They have narrowed it down to three colors: blue, purple, and red.

Here is a summary of what we know:

Voters: Mirabel, Luisa, Dolores, and Isabella.

Candidates: blue, purple, and red.

Personal rankings:

Mirabel: Blue > Purple > Red.

Luisa: Blue > Red > Purple.

Dolores: Red > Purple > Blue.

Isabella: Purple > Blue > Red.

General positional voting system: Each voter gives 1 point to their favorite candidate, s points to their second favorite candidate, and 0 points to their third favorite candidate, where $1 \geq s \geq 0$. We can write this voting system as $[1,s,0]$.

Question 1: Can you find a positional voting system where Blue and Purple tie for first place?

The following Voter Table might be helpful:

	Blue	Purple	Red
Mirabel			
Luisa			
Dolores			
Isabella			

Question 2: Is it possible for there to be an s where Red wins the election? Justify your answer.

The following Voter Table might be helpful:

	Blue	Purple	Red
Mirabel			
Luisa			
Dolores			
Isabella			

Voting system: Each voter gives 10 points to their favorite candidate and 0 points to their least favorite candidate. Each voter gives their second favorite candidate some fixed number of points between 0 and 10. For example, each voter may be asked to give their second favorite candidate 5 points.

Question 1: How many points can the voters give their second favorite candidate so that Purple and Blue tie?

The following Voter Table might be helpful:

	Blue	Purple	Red
Mirabel			
Luisa			
Dolores			
Isabella			

Question 2: Is it possible for Red to win the election? Justify your answer?

The following Voter Table might be helpful:

	Blue	Purple	Red
Mirabel			
Luisa			
Dolores			
Isabella			

Ruth on a road trip

Ruth, her brother Ernie, and their parents are on a road trip, and they are trying to decide on a music genre to listen to. They are deciding between Pop music, Jazz music, Rock music, and Country music. The family will use a Borda count vote to decide on the music genre. If there is a tie for first place, then, because mom is the driver, she will be the tiebreaker.

Ruth knows her family well enough to guess the personal rankings of the music genres that her family members will give.

Here are the personal rankings, based on Ruth's best judgement:

Mom: Jazz > Country > Rock > Pop.

Dad: Country > Rock > Jazz > Pop.

Ernie: Pop > Jazz > Rock > Country.

Ruth: Pop > Rock > Jazz > Country.

Task: How should Ruth vote in the election to get the best musical enjoyment possible for herself?

If you want, you can use the following table.

	Jazz	Country	Rock	Pop
Mom				
Dad				
Ernie				
Ruth				